UNCERTAINTY
IN CONTACT ANGLE ESTIMATES FROM A WILHEMY TENSIOMETER

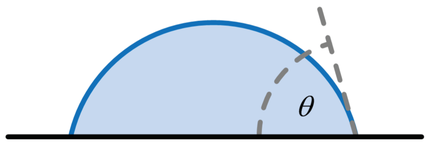
Chuck Extrand, PhD
Director of Research - CPC
St. Paul, MN 55114, USA
chuck.extrand@cpcworldwide.com
+1-651-999-1859

INTRODUCTION

When measuring contact angles (θ) with a tensiometer, how much uncertainty is there in the estimate?

OBJECTIVE

Determine the uncertainty in the estimates of θ from a Wilhelmy tensiometer across the full range of wettability



ANALYSIS

Contact angle (θ) from solid perimeter (p), liquid surface tension (γ) and from the force measured by tensiometer (f),

$$\theta = ArcCos\left(\frac{f}{p\gamma}\right)$$

Estimate uncertainty in contact angles ($\delta\theta$) using standard error propagation techniques,

$$\delta\theta = \left[\left(\frac{\partial\theta}{\partial f}\right)^2(\delta f)^2 + \left(\frac{\partial\theta}{\partial p}\right)^2(\delta p)^2 + \left(\frac{\partial\theta}{\partial\gamma}\right)^2(\delta\gamma)^2\right]^{1/2}$$

where absolute uncertainties are δf, δp and $\delta\gamma$ and corresponding relative uncertainties are

$$\Delta_f = \frac{\delta f}{f} \qquad \Delta_p = \frac{\delta p}{p} \qquad \Delta_\gamma = \frac{\delta\gamma}{\gamma}$$

Insert partial derivatives and rearrange terms

$$\delta\theta = \frac{f/p\gamma}{\left[1-(f/p\gamma)^2\right]^{1/2}}\left(\Delta_f^2 + \Delta_p^2 + \Delta_\gamma^2\right)^{1/2}$$

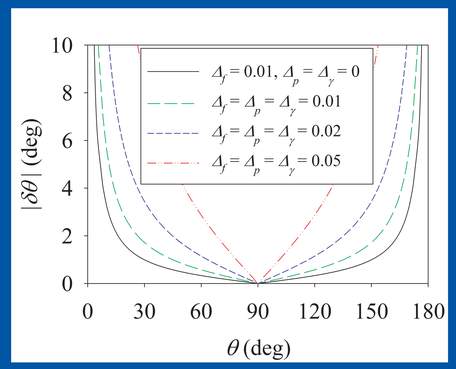
Replace the $f/p\gamma$ terms with $cos\theta$

$$\delta\theta = \pm\cot\theta\cdot\left(\Delta_f^2 + \Delta_p^2 + \Delta_\gamma^2\right)^{1/2}$$

RESULTS

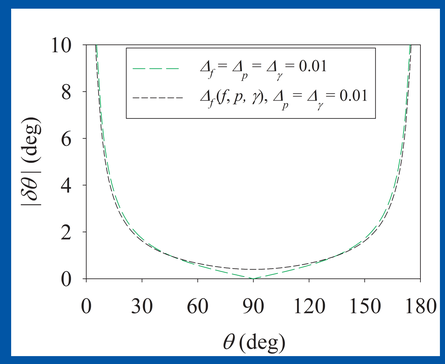
More general result

$$\delta\theta = \pm\cot\theta\cdot\left(\Delta_f^2 + \Delta_p^2 + \Delta_\gamma^2\right)^{1/2}$$



More correct result

$$\delta\theta = \pm\cot\theta\cdot\left[\left(\frac{\delta f}{p\gamma\cdot\cos\theta}\right)^2 + \Delta_L^2 + \Delta_\gamma^2\right]^{1/2}$$



CONCLUSION

Uncertainty in θ from the Wilhelmy tensiometer:

- Small in the middle of the range
- Increases asymptotically at the extremes

CPC™
A DOVER COMPANY