

UNCERTAINTY IN CONTACT ANGLE ESTIMATES FROM THE TANGENT METHOD

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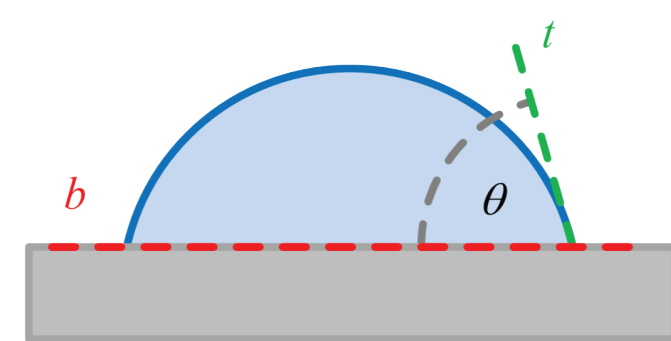
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INTRODUCTION

When measuring contact angles (θ) by the tangent method, how much uncertainty is there?



OBJECTIVE

Determine the uncertainty in measurements of θ from the tangent method across the full range of wettability, from $\theta = 0^\circ$ to $\theta = 180^\circ$

ANALYSIS

Contact angle (θ) from slopes of the baseline (m_b) and tangent line (m_t)

$$\theta = \text{ArcTan} \left| \frac{m_t - m_b}{1 + m_t \cdot m_b} \right|$$

Estimate uncertainty in contact angles ($\delta\theta$) using standard error propagation techniques,

$$\delta\theta = \left[\left(\frac{\partial\theta}{\partial m_t} \right)^2 (\delta m_t)^2 + \left(\frac{\partial\theta}{\partial m_b} \right)^2 (\delta m_b)^2 \right]^{1/2}$$

where absolute uncertainties are δm_b and δm_t and corresponding relative uncertainties are

$$\Delta_t = \frac{\delta m_t}{m_t} \quad \Delta_b = \frac{\delta m_b}{m_t} \quad \text{Taylor, An Introduction to Error Analysis, 1997}$$

Insert partial derivatives and rearrange terms

$$\delta\theta = \frac{1}{1 + m_t^2} \left[(\delta m_t)^2 + (\delta m_b)^2 (1 + m_t^2)^2 \right]^{1/2}$$

Contact angle (θ) is related directly to the slope of the tangent line (m_t) through the tangent function,

$$|m_t| = \text{Tan}\theta$$

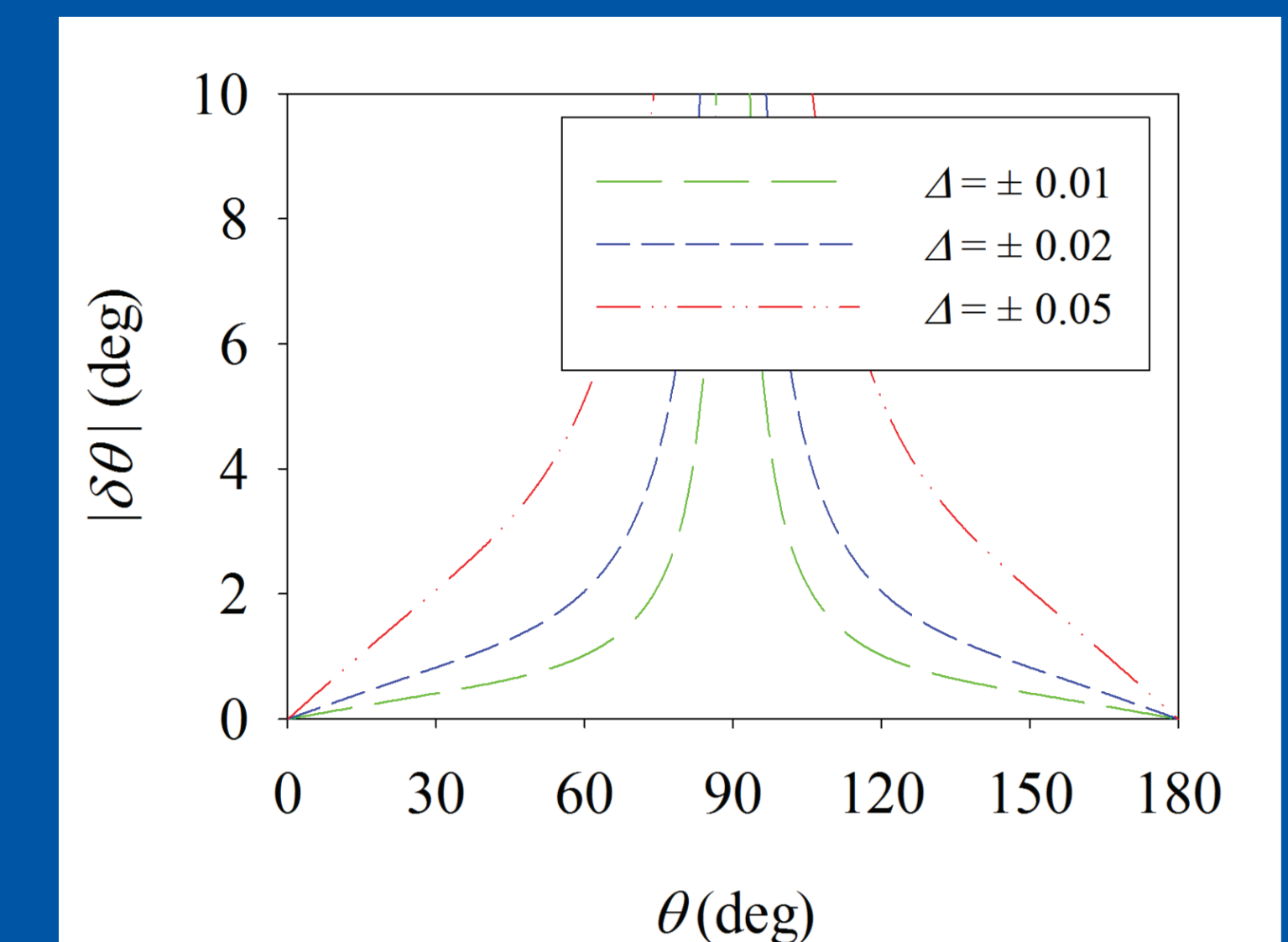
Further simplify relative uncertainties

$$\Delta = \Delta_t = \Delta_b = \frac{\delta m}{m_t}$$

Recast $\delta\theta$ as

$$\delta\theta = \text{Tan}\theta \left[1 + \text{Cos}^4\theta \right]^{1/2} \Delta$$

RESULTS



CONCLUSION

Uncertainty in θ from the tangent method:
Small for low to moderate θ values
Increases asymptotically near $\theta = 90^\circ$